LANDMARK UNIVERSITY, OMU-ARAN

LECTURE NOTE 2
COLLEGE: COLLEGE OF SCIENCE AND ENGINEERING
DEPARTMENT: MECHANICAL ENGINEERING
Course code: MCE 311
Course title: **Applied Thermodynamics**
Credit unit: *3 UNITS.*
Course status: *compulsory*

HEAT ENERGY

This chapter introduces heat energy by looking at the specific heat and latent heat of solids and gases. This provides the base knowledge required for many ordinary estimations of heat energy quantities in heating and cooling, such as are involved in many industrial processes, and in the production of steam from ice and water. The special case of the specific heats of gases is covered, which is important in later chapters, and an introduction is made in relating heat energy to power.

Specific heat

The *specific heat* of a substance is the heat energy required to raise the temperature of unit mass of the substance by one degree. In terms of the quantities involved, the specific heat of a substance is the heat energy required to raise the temperature of l kg of the material by 1°C (or K, since they have the same interval on the temperature scale). The units of specific heat are therefore J/kgK. Different substances have different specific heats, for instance copper is 390 J/kgK and cast iron is 500 J/kgK. In practice this means that if you wish to increase the temperature of a lump of iron it would require more heat energy to do it than if it was a lump of copper of the same mass. Alternatively, you could say the iron 'soaks up' more heat energy for a given rise in temperature.

- Remember that heat energy is measured in joules or kilojoules (1000 joules).
- The only difference between the kelvin and the centigrade temperature scales is where they start from. Kelvin starts at –273 (absolute zero) and centigrade starts at 0. A degree change is the same for each.

The equation for calculating heat energy required to heat a solid is therefore the mass to be heated multiplied by the specific heat of the substance, *c*, available in tables, multiplied by the number of degrees rise in temperature, δT.

$Q = m.c.\delta T$

Putting in the units,

$kJ = kg\ x\ \frac{kJ}{kg.K}\ x\ K$

Note that on the right-hand side, the kg and K terms cancel to leave kJ. It is useful to do a units check on all formulas you use.

Important notes.

- The specific heat of a substance varies depending on its temperature, but the difference is very small and can be neglected.
- You can find the specific heat of a substance by applying a known quantity of heat energy to a known mass of the substance and recording the temperature rise. This would need to be done under laboratory conditions to achieve an accurate answer.
- A body will give out the same quantity of heat energy in cooling as it requires in heating up over the same temperature range.

Specific heats of common substances can be found on data sheets and in reference books, and the values used in the calculations here are realistic.

Example 1

The boiler in a canteen contains 6 kg of water at 20°C. How much heat energy is required to raise the temperature of the water to 100°C? Specific heat of water = 4190 J/kgK.

Solution

$Q = m.c.\delta T$

$Q = 6\ x\ 4190\ x\ (100 - 20)$

Q = 2011200J = 2011.2kJ

Example 2

How many kilograms of copper can be raised from 15°C to 60°C by the absorption of 80 kJ of heat energy? Specific heat of copper = 390 kJ/kgK.

$Q = m.c.\delta T$

$80000 = m\ x\ 390\ x\ (60 - 15)$

$m = \frac{80000}{390\ x\ 45} = \ 4.56kg$

Home Problems**.**

(1) Calculate the heat energy required to raise the temperature of 30 kg of copper from 12°C to 70°C. Assume the specific heat of copper to be 390 J/kgK.

(2) A body of mass 1000 kg absorbs 90 000 kJ of heat energy. If the temperature of the body rises by 180°C, calculate the specific heat of the material of the body.

Power

It is not always useful to know only how much heat energy is needed to raise the temperature of a body. For instance, if you are boiling a kettle, you are more interested in how long it will be before you can make the tea. The quantity of heat energy needed has to be related to the *power* available, in this case the rating of the heating element of the kettle, and if you have a typical kettle of, say, 2kW, it means that in 1 second it provides 2000 joules of heat energy. Remember that power is the rate at which the energy is delivered, i.e. work, or heat energy delivered, divided by time taken. Let us say the kettle contains 2 kg of water and is at a room temperature of 18°C, and the kettle is 2kW. Specific heat of water =
4.2 kJ/kgK.

$Q = m.c.\delta T$

$Q = 2\ x\ 4.2\ x\ (100 - 18) = 688.8kJ$

This is the heat energy required to raise the temperature of the water to 100°C. The kettle is producing 2kW, i.e. 2 kJ/s. Therefore,

$time = \frac{Q}{p} = \frac{688.8}{2} = 344.4\ \text{sec} = 5.74min$ to boil.

The specific heats of gases

Solids have a value of specific heat which varies only slightly with temperature. On the other hand, gases can have many different values of specific heat depending on what happens to it while it is being heated or cooled. The two values which are used are the specific heat at constant pressure, c_p, and the specific heat at constant volume, c_v.
See Figure 1.

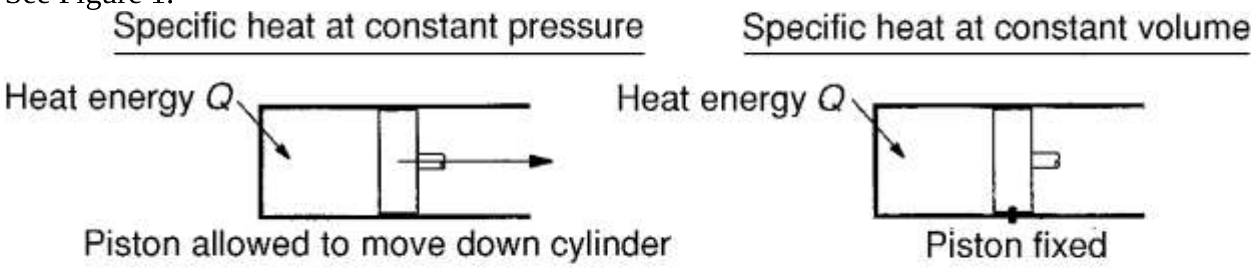

Figure 1 *Specific heats of gases*.

Specific heat at constant pressure, c_p. This is the quantity of heat energy supplied to raise 1 kg of the gas through 1°C or K, while the gas is at constant pressure.
Think of 1 kg of gas trapped in a cylinder. As heat energy is added, the pressure will rise. If the piston is allowed to move down the cylinder to prevent the rise in pressure, the amount of heat energy supplied to raise the temperature of the gas by 1°C is the specific heat of the gas at constant pressure.
Specific heat at constant volume, c_v. This is the quantity of heat energy supplied to raise 1 kg of the gas through 1°C or K, while the gas is at constant volume. Thinking again of the gas in the cylinder, in this case the heat energy is supplied while the piston is fixed, i.e. the volume is constant. The amount of heat energy added for the temperature to rise 1°C is the specific heat at constant volume, c_v.
The specific heat of a gas at constant pressure is always a higher value than the specific heat at constant volume, because when the gas is receiving heat it must be allowed to expand to prevent a rise in pressure, and, while expanding, the gas is doing work in driving the piston down the cylinder. Extra heat energy must be supplied equivalent to the work done.
Note that this is an example of the equivalence of heat energy and work energy.

Example 3

1.5 kg of gas at 20°C is contained in a cylinder and heated to 75°C while the volume remains constant. Calculate the heat energy supplied if c_v = 700 J/kgK.

$Q = m.c_v.\delta T$

$Q = 1.5\ x\ 700\ x\ (75 - 20) = 57750J = 57.75kJ$

Example 4

A gas with a specific heat at constant pressure, c_p = 900 J/kgK, is supplied with 80 kJ of heat energy. If the mass of the gas is 2 kg and its initial temperature is 10°C, find the final temperature of the gas if it is heated at constant pressure.
Solution.

$Q = m.c_p.\delta T$

$80000 = 2\ x\ 900\ x\ \delta T$
$\delta T = \frac{80000}{1800} = 44.44°C$
Final temperature = 10 + 44.44 = 54.44°C

Latent heat

Latent means ‘hidden’, and is used in this connection because, despite the addition of heat energy, no rise in temperature occurs.
This phenomenon occurs when a solid is turning into a liquid and when a liquid is turning into a gas, i.e. whenever there is a ‘change in state’.
In the first case, the heat energy supplied is called the latent heat of fusion, and in the second case it is called the latent heat of vaporization.
The best example to use is water. A lump of ice at, say, –5°C, will need to receive heat energy (sometimes called sensible heat because in this case the temperature does change) to reach 0°C. It will then need latent heat to change state, or melt, during which time its temperature will stay at 0°C.
Further sensible heat energy will then be needed to raise its temperature to boiling point, followed by more latent heat (of vaporization) to change it into steam. See Figure 2.
Just as each substance has its own value of specific heat, so each substance has a value of latent heat of fusion and latent heat of vaporization.
The latent heat of fusion of ice is 335 kJ/kg, that is, it needs 335 kJ for each kg to change it from ice to water. Note that there is no temperature term in the unit because, as we have already seen, no temperature change occurs. Compare this with the unit for specific heat (kJ/kgK).

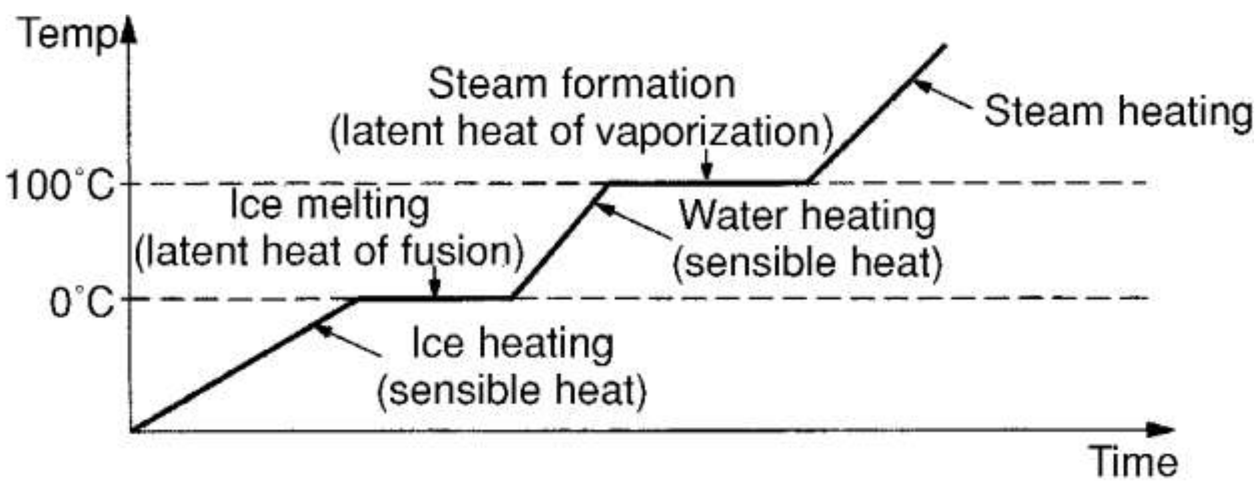

Figure 2 *Sensible and latent heat.*
The latent heat of vaporization of water at atmospheric pressure is 2256.7 kJ/kg. If instead of boiling the water, we are condensing it, we would need to extract (in a condenser) 2256.7 kJ/kg. The values vary with pressure.
The significance of this theory cannot be underestimated, since it relates directly to steam plant and refrigeration plant, both of which we look at later.
Example 5
Calculate the heat energy required to change 4 kg of ice at –10°C to steam at atmospheric pressure.

Specific heat of ice = 2.04 kJ/kgK
Latent heat of fusion of ice = 335 kJ/kg
Specific heat of water = 4.2 kJ/kgK
Latent heat of vaporization of water = 2256.7 kJ/kg
Solution
Referring to Figure 2 we can see that all we have to do is add four values together, i.e. the heat energy to raise the temperature of the ice to 0°C, to turn the ice into water, to raise the water to 100°C, and to change the water at 100°C into steam.
To heat the ice,
$Q_1 = m.c_{ice}\ \delta T$
$Q_1 = 4\ x\ 2.04\ x\ 10 = 81.6kJ$
To change the ice at 0°C into water at 0°C,
$Q_2 = m\ x\ latent\ heat\ of\ fusion_{ice}$
$Q_2 = 4\ x\ 334 = 1340kJ$
To heat the water to 100°C,
$Q_3 = m.c_{water}\ \delta T$
$Q_3 = 4\ x\ 4.2\ x\ 100 = 1680kJ$
To change the water into steam at 100°C,
$Q_4 = m\ x\ latent\ heat\ of\ vaporization_{water}$
$Q_4 = 4\ x\ 2256.7 = 9026kJ$
Total heat energy = $Q1$ + $Q2$ + $Q3$ + $Q4$
= 81.6 + 1340 + 1680 + 9026.8
= 12 128.4 kJ.
In this example, we could provide the 12 128.4 kJ very quickly with a large kW heater, or much more slowly with a small kW heater.
Neglecting losses, the result would be the same, i.e. steam would be produced. As an exercise, and referring to the earlier example of the kettle, find how long it would take to produce the steam in Example 5 if you used a 2 kW heater and then a 7 kW heater, neglecting losses. You will notice that we have dealt here mainly with water, since this is by far the most important substance with which engineers must deal. The theory concerning the heating of water to produce steam is the same as for the heating of liquid refrigerant in a refrigeration plant.
Home Problems 2
(1) Calculate the quantity of heat energy which must be transferred to 2.25 kg of brass to raise its temperature from 20°C to 240°C if the specific heat of brass is 394 J/kgK.
(2) Find the change in temperature produced by 10 kJ of heat energy added to 500 g of copper. Specific heat of copper = 0.39 kJ/kgK.
(3) Explain why, for a gas, the specific heat at constant volume has a different value from the specific heat at constant pressure.
An ideal gas is contained in a cylinder fitted with a piston. Initially the temperature of the gas is 15°C. If the mass of the gas is 0.035 kg, calculate the quantity of heat energy required to raise the temperature of the gas to 150°C
when:
(a) the piston is fixed;
(b) the piston moves and the pressure is constant.
For the gas, c_v = 676 J/kgK and c_p = 952 J/kgK.

Work done and heat energy supplied.

We have looked at gas processes and how to calculate pressures, temperatures and volumes after a process has occurred. This is very important in, for instance, engine design, where important parameters are piston and cylinder size and combustion temperature. We can now go on to find work and heat energy transfer occurring during a process, which prepares us for the next chapter in which these elements are put together to form complete engine cycles.

Using the *p/V* diagram to find work

We have seen the usefulness of the *p*/*V* diagram in visualizing the gas processes and showing the variables. They are also important in allowing us to find expressions for the work done during a process. To demonstrate this, we will look at the constant pressure process. Figure 1 shows the *p*/*V* diagram as the piston moves down the cylinder with no change in pressure.

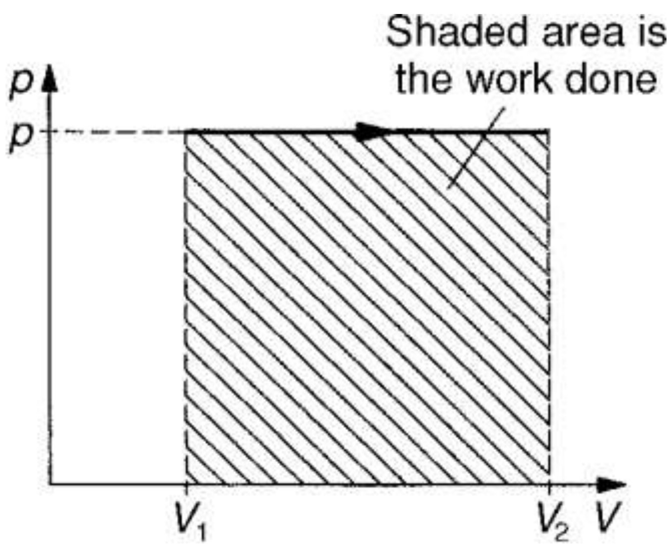

Figure 2.3.1 *Constant pressure process*

The area of the rectangle under the constant pressure line is its height multiplied by its breadth, i.e. $p(V_2 - V_1)$.

Putting in the units:

$P = \frac{N}{m^2}$

$V = m^3$

Area of rectangle = $P(V_2 - V_1) = \frac{N}{m^2}\ x\ m^3 = N.m$

1(N.m) =1(joule)

- Hence, by finding the area under the *p*/*V* diagram, we have found the work done during the process.

We now have an expression for the work done during a constant pressure process, and by looking at the other processes on the *p*/*V* diagram and finding the area under each, we will have all the expressions for work done. Unfortunately, the polytropic, adiabatic and isothermal processes are, as we have seen, curves on the *p*/*V* diagram, and to find these areas we must use calculus.

Expressions for work done

Constant volume

$W = 0$

This must be so because on the *p*/*V* diagram the process is a straight vertical line which has no area beneath it, and because the piston does not move.

Constant pressure

$W = p.\delta V$

Adiabatic

$$W = \left(\frac{P_1V_1 - P_2V_2}{\gamma - 1}\right)$$

Polytropic

$$W = \left(\frac{P_1V_1 - P_2V_2}{n - 1}\right)$$

Isothermal

$$W = P_1V_1 ln\left(\frac{V_2}{V_1}\right)$$

Note: ln is shorthand for $\log_e$.

- Using these expressions, we can find work done during any gas process.

In reality, almost all practical processes are polytropic, i.e. $pV^n = c$. The adiabatic and isothermal processes are impossible to achieve, but useful as 'ideal' processes.

In an engine, the non-flow expansions and compressions are all polytropic, meaning that heat energy is transferred through the cylinder walls, and pressure, volume and temperature all change.

The same is true in flow processes such as gas turbines where the same equations can be used across inlet and outlet conditions.

Important mathematics.

Finding areas under curves is a common task in mathematics and an extremely useful facility for engineers. We can find areas by using Simpson's rule and other similar methods, or simply by plotting on graph paper and counting the number of squares. But the best method – if the equation of the curve is known – is to use calculus, which leads to an equation which gives a precise answer.

In this example, we are going to find the area under a curve of the form pV^n = Constant. This, as we have seen, is the relationship between *p* and *V* when a polytropic process occurs, in other words what is happening in almost all engines during expansion and compression of the gases. It gives the expression for work done during a polytropic process.

Figure 2 shows the *p*/*V* diagram for a polytropic compression. We carry out the usual procedure for integration which is to consider a strip of the area we want to find, and add up all the strips in the area to give an expression for the total area.

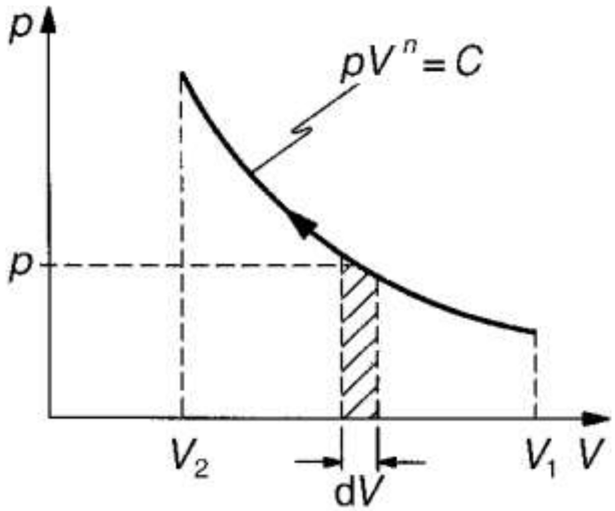

Figure 2 *Polytropic work done*

Area of strip = P.dV

Total area $= \int_{V_1}^{V_2} P.dV$

From $P.V^n = C$, $P = \frac{C}{V^n}$

Total area $= \int_{V_1}^{V_2} . \frac{C}{V^n} dV$

Taking c outside and putting V^n at the top,

Area $= c\int_{V_1}^{V_2} V^{-n} dV$

Integrating,

$$A = c\left[\frac{V^{1-n}}{1-n}\right]_{V_1}^{V_2}$$

substituting V_1 and V_2,

$$A = \frac{c}{1-n}\left[V_2^{1-n}\, 1 - V_1^{1-n}\right]$$

$$= \frac{1}{1-n}\left[\frac{c}{V_2^n}V_2 - \frac{c}{V_1^n}V_1\right]$$

Substituting $P_1 = \frac{c}{V_1^n}$ and $P_2 = \frac{c}{V_2^n}$

$$A = \frac{c}{1-n}\left[P_2V_2 - P_1V_1\right]$$

It is more convenient to write $(1 - n)$ as $(n - 1)$, giving,

$$A = \frac{[P_2V_2 - P_1V_1]}{n-1} = Polytropic\ work\ done.$$

Example 1

A piston moves down the cylinder in an engine from top dead centre to bottom dead centre, while the pressure in the cylinder remains constant at 30 bar. The clearance volume is 0.002m^3 and the swept volume is 0.05m^3. What is the work done?

The process is shown in Figure 3.

$P = 30$ bar $= 30 \times 10^5\ \text{N/m}^2$

V_1 = clearance volume = 0.002m^3

V_2 = swept volume + clearance volume = 0.002 + 0.05 = 0.052m^3

In a constant pressure process,

Work done = $p(V_2 - V_1)$

$30 \times 10^5 \times (0.052 - 0.002) = 150\,000$ J = 150 kJ

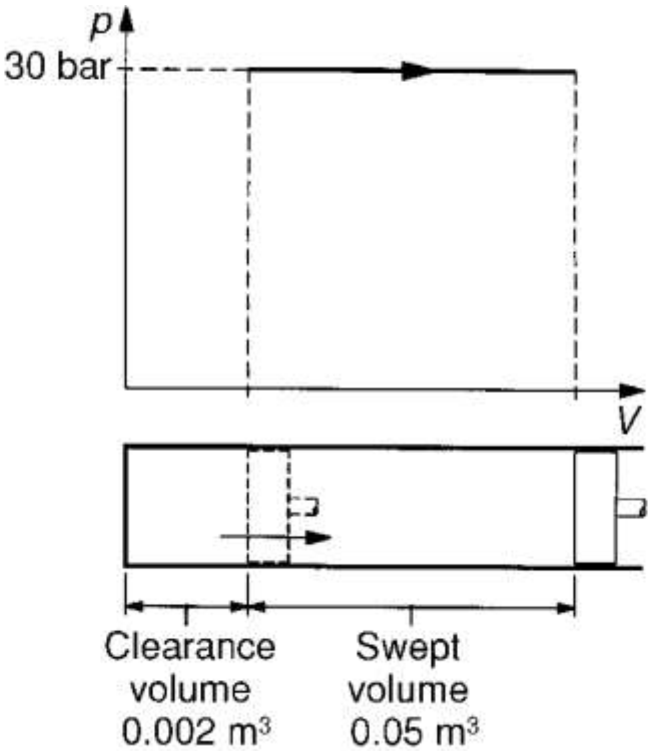

Figure 3 *Example 1.*

As an example of how we could calculate a value of power produced, if we assume in this example (forgetting for the moment that a constant pressure expansion in an engine cylinder is in practice highly unlikely), that the piston is forced down the cylinder every revolution – as it would be in a two-stroke engine – it would mean that if the engine was turning at 300 rpm then 300 x 150 = 45 000 J of work would be done each minute.

This is 750 J/s = 750W, the power produced in the cylinder.

Note.

In the expressions for work done, pressure must be in N/m^2 or kN/m^2, giving answers in J and kJ respectively.

Example 2

The bore and stroke of the cylinder of a diesel generator are 146 mm and 280 mm respectively, and the clearance volume is 6% of the swept volume. If the air before compression is at 1 bar, find the work done in compression if the process is:

(a) polytropic, *n* = 1.3;

(b) isothermal.

$$Swept\ volume = \frac{\pi\ x\ (bore)^2}{4}\ x\ stroke$$

$$= \frac{\pi\ x\ 0.146^2}{4}\ x\ 0.28 = 0.00469m^3$$

Clearance volume = 0.06 x swept volume

= 0.06 x 0.00469 = 0.000 28m^3

Initial volume = swept volume + clearance volume

= 0.00469 + 0.000 28

= 0.00497m^3

Polytropic compression

$P_1V_1^n = P_2V_2^n = 1\,x\,0.00497^{1.3} = P_2\,x\,0.00028^{1.3}$

$P_2 = 1\,x\,\left(\frac{0.00497}{0.00028}\right)^{1.3} = 42.1\,bar$

$W = \left(\frac{P_1V_1 - P_2V_2}{n-1}\right)$

$= \frac{10^5[(1\,x\,0.00497) - (42.1\,x\,0.00028)]}{1.3-1} = 2273J = 2.273kJ$

Isothermal compression

$W = P_1V_1 ln\left(\frac{V_2}{V_1}\right)$

$= 1\,x\,10^5\,x\,0.00497\,x\,ln\left(\frac{0.00028}{0.00497}\right) = 1430J = 1.430kJ$

Note that least work is done when the compression is isothermal, and in the case of an air compressor for instance, isothermal compression is desirable in order to reduce the power input necessary from its engine or motor. An approximation to isothermal compression is achieved by multi-stage compression with intercooling.

Alternative expressions for work done

The characteristic gas equation gives us the relationship,

pV = *m.R.T*

You will see that *pV* occurs in the expressions for work done. We can substitute *m.R.T* instead. The expressions for work done then become,

Constant volume

W = 0

Constant pressure

W = *m.R*(*T*1 – *T*2)

Adiabatic

$W = \frac{m.R(T_1 - T_2)}{\gamma - 1}$

Polytropic

$W = \frac{m.R(T_1 - T_2)}{n-1}$

Isothermal

$W = m.R.T_1 ln\left(\frac{V_2}{V_1}\right)$

These expressions are necessary if the values given in a problem do not allow the use of the '*pV*' equations, or simply if they are preferred in cases when either could be used.

Example 3

1.5 kg of gas is compressed isothermally at a temperature of 20°C, from a volume of 0.5m3 to a volume of 0.3m3. If *R* for the gas is 290 J/kgK, find the work done.

We cannot use the '*p.V*' expression for the work done, because the initial pressure is unknown. We could find the pressure using *p.V* = *m.R.T* and then use it, but this is a case where the '*m.R.T*' version can be used directly.

Solution.

$W = m.R.T_1 ln\left(\frac{V_2}{V_1}\right)$

$W = 1.5\,x\,290\,x\,(20 + 273) ln\left(\frac{0.3}{0.5}\right) = -65107J = -65.2kJ$

Note.
The work done is negative, because we have done work *on* the gas. Work done *by* the gas is positive.

Heat energy transferred during a process

If we have a non-flow process occurring, say, within an engine cylinder, then, inevitably, some heat energy is transferred to or from the gas through the cylinder walls. Usually, this heat energy is transferred out of the cylinder walls to the engine cooling water.

The quantity of heat energy transferred affects the values of pressure and temperature achieved by the gas within the cylinder. Only in an adiabatic process is there no transfer of heat energy to or from the gas, and this situation is impossible to achieve in practice.

In order to find how much heat energy has been transferred during a non-flow process, we use the non-flow energy equation (NFEE) (see below, 'The non-flow energy equation'.

$Q = W + (U_2 – U_1)$

In words, this means that the heat energy transferred through the cylinder walls is the work done during the process added to the change in internal energy during the process. Remember that the internal energy of the gas is the energy it has by virtue of its temperature, and that if the temperature of the gas increases, its internal energy increases, and vice versa.

Using the NFEE is straightforward, in that we have already seen the equations for work done, and the change in internal energy for any process (see below, 'Internal energy') is given by

$U_2 – U_1 = m.c_v (T_2 – T_1)$

We can calculate each and add them together.

Applying the NFEE to each process gives,

Constant volume

$Q = W + (U_2 – U_1)$. $W = 0$ in a constant volume process.

$Q = (U_2 – U_1)$

$Q = m.c_v (T_2 – T_1)$

Constant pressure

$Q = W + (U_2 – U_1)$. $W = p.\_V$ in a constant pressure process.

$Q = p(V_2 – V_1) + m.cv (T_2 – T_1)$

It is useful to digress here to establish an important expression concerning the gas constant, R, and values of c_p and c_v.

We know that the heat energy supplied in the constant pressure process is

$Q = m.c_p (T_2 – T_1)$

therefore,

$m.cp (T_2 – T_1) = p(V_2 – V_1) + m.cv (T_2 – T_1)$

Substituting from $p.V = m.R.T$), $p(V_2 – V_1) = m.R(T_2 – T_1)$, and dividing by $(T_2 – T_1)$,

$c_p = R + c_v$ for unit mass

$R = c_p – c_v$

Adiabatic

$Q = W + (U_2 – U_1)$

Q = 0.

This is the definition of an adiabatic process.

Note: if W and δU were calculated and put in the formula, the answer would be 0.

Polytropic

$Q = W + (U_2 – U_1)$

$$Q = \frac{(P_1V_1 - P_2V_2)}{n-1} + m.c_v(T_2 - T_1)$$

Isothermal

$Q = W + (U_2 - U_1)$. No change in temperature, therefore no change in U.

$Q = P_1V_1 ln\left(\frac{V_2}{V_1}\right)$

Example 4

$0.113m^3$ of air at 8.25 bar is expanded in a cylinder until the volume is $0.331m^3$. Calculate the final pressure and the work done if the expansion is polytropic, $n = 1.4$. If the temperature before expansion is 500°C, and $c_v = 245$ J/kgK, find the heat energy transferred during the process. $R = 810$ J/kgK.

The process is shown in Figure 4

Solution.

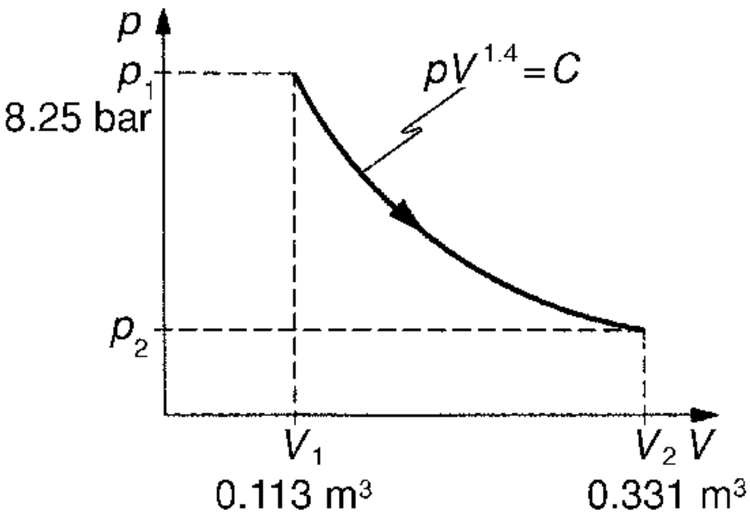

Figure 4 *Example 4.*

$P_1V_1^n = P_2V_2^n$

$8.25\ x\ 0.113^{1.4} = P_2\ x\ 0.331^{1.4}$

$P_2 = 8.25\ x\ \frac{0.113^{1.4}}{0.331^{1.4}} = 8.25\ x\ \left(\frac{0.113}{0.331}\right)^{1.4} = 1.83\ bar$.

$W = \left(\frac{P_1V_1 - P_2V_2}{n-1}\right)$

$= \frac{(8.25\ x\ 10^5\ x\ 0.113) - (1.83\ x\ 10^5\ x\ 0.331)}{1.4-1} = 81630J = 81.63kJ$

To find the heat energy transferred we use the non-flow energy equation,

$Q = W + (U_2 - U_1)$

In this case, $W = 81.63$ kJ, and $(U_2 - U_1) = m.c_v\ (T_2 - T_1)$.

We need to find T_2 and m, the mass of gas in the cylinder.

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{8.25\ x\ 0.113}{(500+273)} = \frac{1.83\ x\ 0.331}{T_2}$

$T_2 = \frac{1.83\ x\ 0.331\ x\ 773}{8.25\ x\ 0.113} = 502.3K$

From $P_1V_1 = m.R.T$

$m = \frac{P_1V_1}{RT_1} = \frac{8.25\ x\ 10^5\ x\ 0.113}{810\ x\ 773} = 0.149kg$

$U_2 - U_1 = m.c\text{v} (T_2 - T_1)$
$= 0.149 \times 245(502.3 - 773)$
$= - 9881.9$ J $= -9.9$ kJ
$Q = W + (U_2 - U_1) = 81.63 + (-9.9) = 71.7$ kJ
For this process to occur with these conditions, 71.7 kJ of heat energy must be supplied to the gas.
Note.
The heat energy transfer is positive because heat energy has been supplied to the gas. Heat energy rejected is negative.
Home Problems
(In all cases, sketch the p/V diagram).
(1) An ideal gas is contained in a cylinder fitted with a piston. Initially the V, p and T are 0.02m^3, 1.4 bar and 15°C respectively. Calculate the quantity of heat energy required to raise the temperature of the gas to 150°C when:
(a) the piston is fixed;
(b) the piston moves and the pressure is constant.
$c\text{v} = 676$ J/kgK
$c\text{p} = 952$ J/kgK
(2) 0.25m3 of gas is compressed isothermally from a pressure of 1 bar until its volume is 0.031m^3. Calculate the heat energy transfer.
(3) 1m3 of gas at 10 bar is expanded polytropically with an index of $n = 1.322$ until its volume is doubled. The final pressure is 4 bar. Calculate the work done during the process.
(4) Air expands adiabatically, $\gamma = 1.4$, from 12 bar, 50°C to 4 bar. Calculate the final temperature and the work done per kg of air if the expansion takes place in a cylinder fitted with a piston. R = 287 J/kgK.
(5) A perfect gas of mass 0.2 kg at 3.1 bar and 30°C is compressed according to the law $pV1.3 = C$ until the pressure is 23.54 bar, temperature 210°C. Find the initial volume, the final volume, the work done and the heat energy transferred during the process.
$c_v = 700$ J/kgK
$c_p = 980$ J/kgK
(6) A perfect gas of volume 0.085m3 pressure 27.6 bar and temperature 1800°C is expanded to 3.3 bar and at this pressure its volume is 0.45m^3. If the index of expansion is
1.27, find:
(a) The final temperature;
(b) The work done during the process.

Internal combustion engines

We have seen the non-flow processes of constant volume, constant pressure, adiabatic, polytropic and isothermal expansion and compression, and have found how to calculate work done and heat energy transferred during these operations. In this chapter we apply this knowledge to theoretical and practical engine cycles, looking at the processes which make up the cycle, the calculation of brake and indicated power, mean effective pressure, efficiency and fuel consumption, and the relationship between theoretical and actual. Also, we examine how power and other parameters can be found in practice.

The second law of thermodynamics

It is useful at this stage to consider the heat engine cycle in general terms.

The *thermal efficiency* will always be given by the ratio of what we get out of the engine in terms of work, to the amount of heat energy supplied. From the first law we know that in a closed cycle, the change in internal energy is zero, and the net-work energy transfer equals the net heat energy transfer. Therefore the work done is the difference between the heat energy supplied and the heat energy rejected, hence,

$$Thermal\ efficiency = \frac{work\ done}{heat\ energy\ supplied}$$

$$= \frac{heat\ supplied - heat\ rejected}{heat\ spplied}$$

$$Thermal\ efficiency = 1 - \frac{heat\ rejected}{heat\ \ supplied}$$

This expression can be applied to all the *ideal cycles* we look at next. Clearly, the efficiency can only be 100% if no heat energy is rejected. We know, and can demonstrate, that it is not possible to construct a heat engine which can operate without rejecting heat energy.

The *second law of thermodynamics* encapsulates this:

'It is impossible for a heat engine to produce work if it exchanges heat energy from a single heat reservoir'

In other words, there must be a 'cold reservoir' to which heat energy is rejected.

The second law has other versions, e.g.

'Wherever a temperature difference occurs, motive power can be produced'

'If a system is taken through a cycle and produces work, it must be exchanging heat with two reservoirs at different temperatures'

The establishment of the second law is due mainly to Sadi Carnot in the nineteenth century.

The Carnot cycle

Carnot proposed a cycle which would give the maximum possible efficiency between temperature limits. Figure 1 shows this cycle which consists of an isothermal expansion ($pV = C$) of the gas in the cylinder from point 1 to point 2 as heat energy is supplied, followed by an adiabatic expansion ($pV\_ = C$) to point 3. Between 2 and 3 the gas has cooled. The piston moves up the cylinder between 3 and 4 compressing the gas isothermally as heat energy is rejected, and between 4 and 1 the gas is compressed adiabatically. All the processes are reversible, and heat energy is supplied and rejected at constant temperature. The cycle is therefore impossible to create in practice. A lengthy proof, using the non-flow energy equation and the expressions for work done substituted into the expression for efficiency we have just derived, gives an expression for the efficiency of the cycle, i.e. the Carnot efficiency.

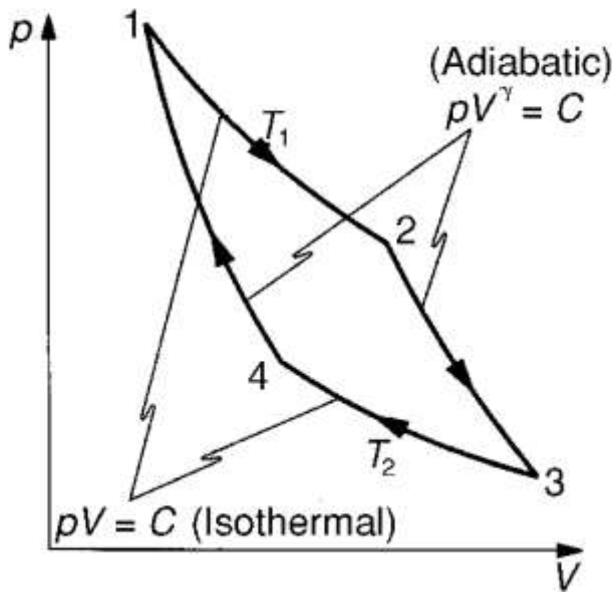

Figure 1 *Carnot cycle*

Carnot efficiency, $\eta = 1 - \frac{T_2}{T_1}$

T_2 is the lowest and T_1 the highest of only two temperatures involved in the cycle. The cycle cannot be created in practice, since we have reversible processes and must supply and reject heat at constant temperature. It does, however, supply a means of rating the effectiveness of a cycle or plant by allowing us to calculate a maximum theoretical efficiency based on maximum and minimum temperatures, even if the cycle is not operating on the Carnot cycle.

Example 1

A diesel engine works between a maximum temperature of 600°C and a minimum temperature of 65°C. What is the Carnot efficiency of the plant?

Carnot efficiency = $\eta = 1 - \frac{T_2}{T_1}$

$= 1 - \frac{(65+273)}{600+273} = 1 - \frac{338}{873} = 0.613 = 61.3\%$

The air standard cycles

In what are known as air standard cycles, or ideal cycles, the constant volume, constant pressure and adiabatic processes are put together to form theoretical engine cycles which we can show on the *p*/*V* diagram. The actual *p*/*V* diagram is different from what is possible in practice, because, for instance, we assume that the gas is air throughout the cycle when in fact it may be combustion gas. We also assume that valves can open and close simultaneously and that expansion and compressions are adiabatic. Air standard cycles are reference cycles which give an approximation to the performance of internal combustion engines.

Constant volume (Otto) cycle

This is the basis of the petrol engine cycle. Figure 2 shows the cycle, made up of an adiabatic compression, 1–2 (piston rises to compress the air in the cylinder), heat energy added at constant volume, 2–3 (the fuel burns), adiabatic expansion, 3–4 (the hot gases drive the piston down the cylinder), and heat energy rejected at constant volume, 4–1 (exhaust).

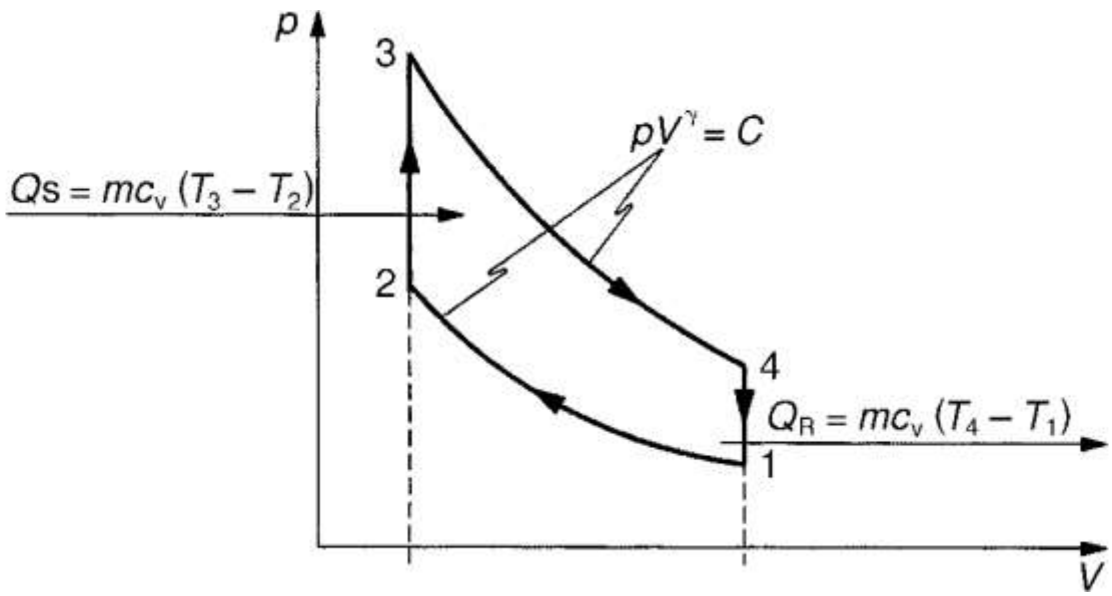

Figure 2 *Constant volume (Otto) cycle*

What can we do with this? We can calculate from our knowledge so far:

- the pressures, volumes and temperatures around the cycle;
- the work done during each of the processes and therefore the net work done;
- the heat energy transferred during each process;
- the ideal – or air standard – efficiency of the cycle using the expression we derived earlier in this chapter.

Indicated mean effective pressure, P_{mi}

The air standard efficiency of the cycle is a useful indicator of actual performance, but limited because it is not easy to decide in practice where heat energy transfers begin and end. The ideal cycle diagram – and an actual indicator diagram, which we see later – can also provide a value of *indicated mean effective pressure*, P_{mi}, which is another useful comparator. This is found by dividing the height of the diagram by its length to produce a rectangle of the same area. See Figure 3, in which the rectangle is shown hatched.

$$P_{mi} = \frac{area\ of\ diagram}{length} = \frac{A}{L}$$

The area of the diagram is work, joules = N.m. The length of the diagram is volume, i.e. m^3.

$$P_{mi} = \frac{N.m}{m^3} = \frac{N}{m^2}$$

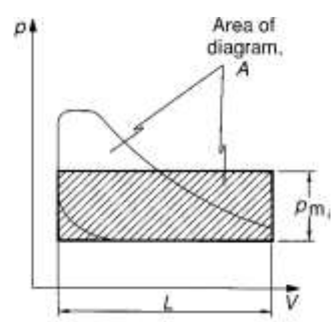

Figure 3 *Mean effective pressure*

Key notes.

- The indicated mean effective pressure can be thought of as the pressure acting on the piston over full stroke which would give the same work output. Generally speaking, the higher this is the better.
- Since the brake power of an engine is usually quoted and easier to record, instead of an indicated mean effective pressure, a value of *brake mean effective pressure*, P_{mb}, is more often used as the comparator. It is found by using the expression for indicated power with values of brake power and brake mean effective pressure substituted. This is covered later in the chapter.

Example 2

The ratio of compression of an engine working on the constant volume cycle is 8.6:1. At the beginning of compression the temperature is 32°C and at the end of heat supply the temperature is 1600°C. If the index of compression and expansion is 1.4, find:

(a) the temperature at the end of compression;

(b) the temperature at the end of expansion;

(c) the air standard efficiency of the cycle.

Figure 4 shows the cycle.

Note:

- the compression ratio is a ratio of volumes, V_1/V_2, not a ratio of pressures;
- the dimensionless ratio values of 8.6 and 1 are used directly in the equations;
- the heat energy transfer in a constant volume process is ($m.c$v.δT).

Solution.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}, \frac{T_2}{305} = \left(\frac{8.6}{1}\right)^{1.4-1}, T_2 = 305\ x\ 8.6^{0.4}$$

= 721.3K, temperature at end of compression.

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}, \frac{T_4}{1873} = \left(\frac{1}{8.6}\right)^{1.4-1}, T_4 = 1873\ x\ 0.42$$

= 792K, temperature attend of expansion.

Air standard efficiency,

$$= 1 - \frac{heat\ rejection}{heat\ supplied}$$

$$= 1 - \frac{m.c_v(T_4 - T_1)}{m.c_v(T_3 - T_2)}$$, m and c_v are cancel

$$= 1 - \frac{792-305}{1873-721.3} = 1 - \frac{487}{1151.7}$$

= 0.577 = 57.7% air standard efficiency.

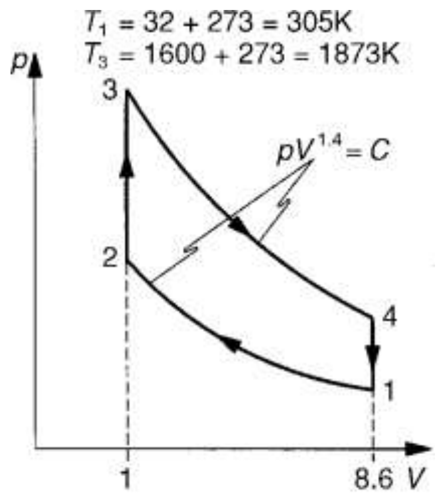

Figure 4 *Example 2*

Example 3

In an air standard (Otto) constant volume cycle, the compression ratio is 8 to 1, and the compression commences at 1 bar, 27°C. The constant volume heat addition is 800 kJ per kg of air. Calculate:

(a) the thermal efficiency;

(b) the indicated mean effective pressure, *P*mi.

*c*v = 718 J/kgK

γ = 1.4

Figure 5 shows the cycle.

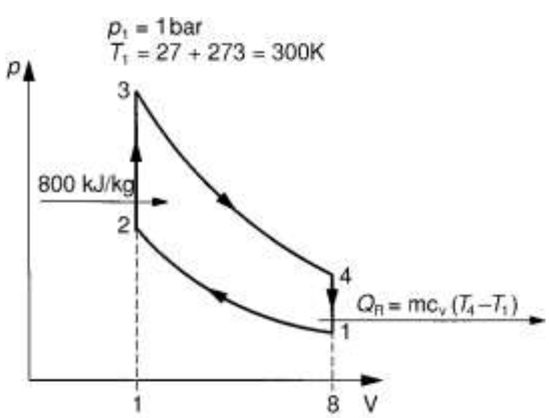

Figure 5 *Example 3*

$$\gamma = \frac{c_p}{c_v}, c_p = \gamma - c_v = 1.4\, x 718$$

= 1005J/kgK, see adiabatic compression and expansion.

$$R = c_p - c_v, R = 1005 - 718 = 287\, J/kgK$$

$$P_1 V_1 = m.R.T_1,\ V_1 = \frac{m.RT_1}{P_1} = \frac{1\ x\ 287\ x\ 300}{1\ x\ 10^5} = 0.861\ m^3$$

$$\frac{V_1}{V_2} = 8, V_2 = \frac{0.861}{8} = 0.1076\ m^3$$

$$Swept\ volume = V_2 - V_1 = 0.861 - 0.1076 = 0.7534 m^3$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}, \frac{T_2}{300} = \left(\frac{8}{1}\right)^{1.4-1}, T_2 = 300\ x\ 8^{0.4} = 689.2K.$$
$$Q_{1-2} = m.c_v(T_3 - T_2)$$
$$T_3 = 689.2 + \frac{800\ x\ 10^3}{718} = 1803.4K \quad (m = 1kg)$$
$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}, T_4 = 1803.4\left(\frac{1}{8}\right)^{0.4} = 785K$$

Heat energy rejected $= Q_{4-1} = m.c_v(T_4 - T_1)$

= 718(785 -300) = 348.2kJ

Work = heat supplied – heat rejected

= 800 – 348.2 = 451.8 kJ

Air standard efficiency = $\eta = 1 - \frac{heat\ rejection}{heat\ supplied}$

$$1 - \frac{451}{800} = 1 - 0.5647 = 0.435 = 43.5\%$$

Mean effective pressure, $P_{mi} = \frac{area\ of\ diagram}{length}$

$= \frac{work}{Swep\ volume}$

$= \frac{451.8}{0.7534} = 599.7\ kN/m^2$

Constant pressure (diesel) cycle

This is the basis of some diesel engine cycles. Figure 6 shows the cycle which consists of adiabatic compression, 1-2, constant pressure heat addition, 2-3, adiabatic expansion, 3-4 and exhaust at constant volume, 4-1

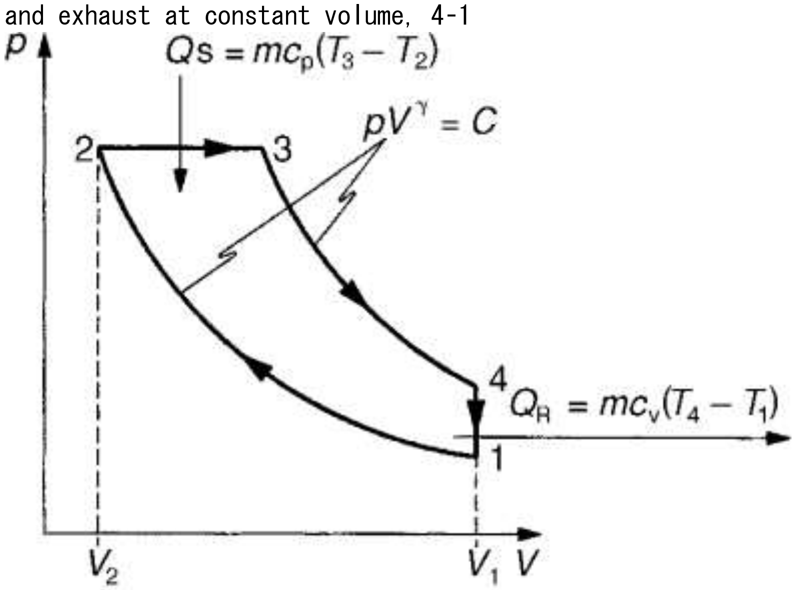

Figure 6 *Constant pressure (diesel) cycle*

Example 4

In an air standard diesel cycle, the compression commences at 1 bar, 27°C. Maximum pressure is 45 bar and the volume doubles during the constant pressure process. Calculate the air standard efficiency.

Solution.

For air;

$\gamma = 1.4, c_p = 1005\ J/kgK., c_v = 718J/kgK$

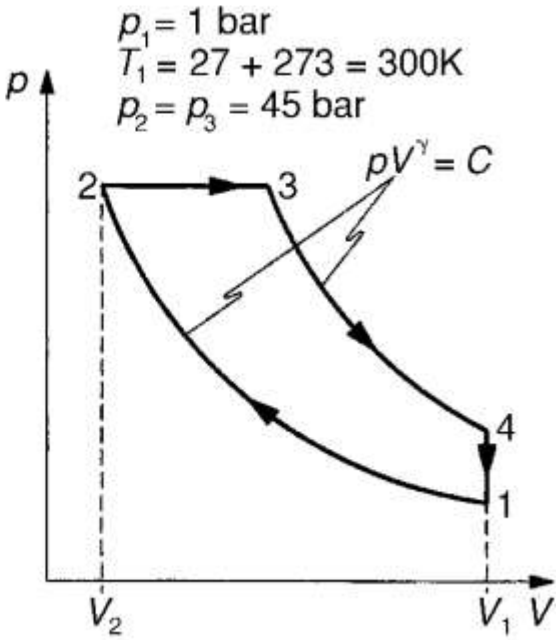

Figure 7 *Example 4.*

Figure 7 shows the cycle.
$R = c_p - c_v = 1005 - 718 = 287$ J/kgK
Using a mass of 1 kg,
$p_1 V_1 = m.R.T_1$

$$V_1 = \frac{m.R.T_1}{P_1} = \frac{1\,x\,287\,x\,(27+273)}{1\,x\,10^5} = 0.861m^3$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}, T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 300\left(\frac{45}{1}\right)^{0.286} = 891K$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{P_2T_1} = \frac{1\,x\,0.861\,x\,891}{45\,x\,300} = 0.0568m^3$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}, \qquad p_2\ and\ P_3\ are\ cancel$$

$$T_3 = \frac{V_3T_2}{V_2} = \frac{(0.0568\,x\,2)\,x\,891}{0.0568} = 1782K$$

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = 1780\left(\frac{2\,x\,0.0568}{0.861}\right)^{0.4} = 791.7K$$

$$Air\ standard\ efficiency = 1 - \frac{heat\ rejected}{heat\ supplied}$$

$$= 1 - \frac{m.c_v(T_4 - T_1)}{m.c_p(T_3 - T_2)} = 1 - \frac{718(791.7-300)}{1005(1780-891)} = 0.605 = 60.5\%\ air\ standard\ efficiency.$$

(Cancel the m’s.).

Mixed pressure (dual combustion) cycle

This is the basis of most diesel engine cycles. Figure 8 shows the cycle, consisting of heat addition at constant volume *and* at constant pressure, two adiabatics and constant volume heat rejection. Note the expressions for heat addition and rejection on the diagram.

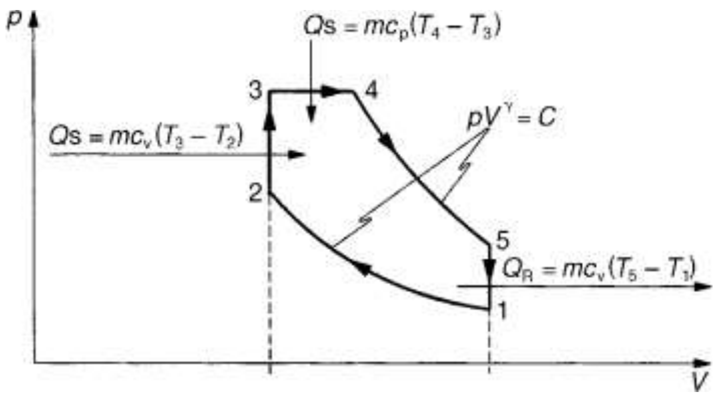

Figure 8 *Mixed pressure (dual combustion) cycle*

Example 5

A compression ignition engine working on the ideal dual combustion cycle has a compression ratio of 16:1. The pressure and temperature at the beginning of compression are 98 kN/m^2 and 30°C respectively. The pressure and temperature at the completion of heat supplied are 60 bar and 1300°C. Calculate the thermal efficiency of the cycle.

c_v = 717 J/kgK

c_p = 1004 J/kgK

Solution.

Figure 9 shows the cycle.

$p_1 = 98$ kN/m^2

$T_1 = 30 + 273 = 303$K

$V_1 = V_5 = 16$

$V_2 - V_3 = 1$

$p_3 = p_4 = 60$ bar

$T_4 = 1300 + 273 = 1573$K

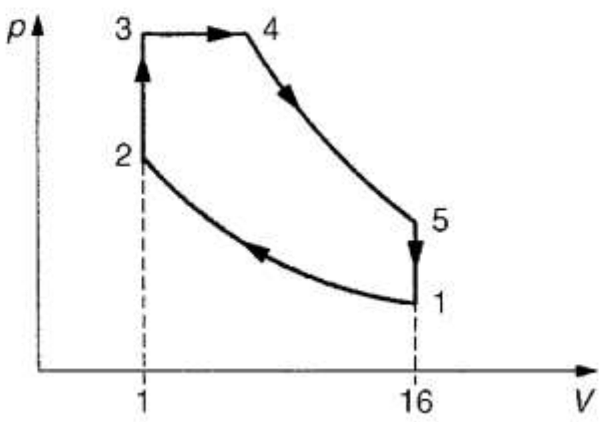

Figure 9 *Example 5.*

$$\gamma = \frac{c_p}{c_v} = \frac{1004}{717} = 1.4$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 0.98\left(\frac{16}{1}\right)^{1.4} = 47.53\ bar$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 303\left(\frac{16}{1}\right)^{0.4} = 918K$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}, (cancel\ V_2), T_3 = \frac{60\ x\ 918}{47.53} = 1159K$$
$$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4}, (cancel\ P_3), V_4 = \frac{V_3\ x\ 1573}{1159} = 1.358V_3$$
$$P_4V_4^{\gamma} = P_5V_5^{\gamma}$$
$$P_5 = P_4\left(\frac{V_4}{V_5}\right)^{\gamma} = 60\left(\frac{1.358V_3}{16V_3}\right)^{1.4} = 1.899bar$$
$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = 1573\left(\frac{1.358V_c}{16V_c}\right)^{0.4} = 586.5K$$
Heat energy supplied = $m.c_v(T_3 - T_2) + m.c_p(T_4 - T_3)$
= 0.717(1159 – 918) + 1.004(1573 – 1159), using a mass of 1kg
= 173 + 416 = 589 kJ/kg
Heat energy rejected = $m.c_v(T_5 - T_1)$
= 0.717 (586.5 – 303)
= 203.3 kJ/kg
$$Air\ standard\ efficiency = 1 - \frac{heat\ rejected}{heat\ supplied}$$
$$= 1 - \frac{203.3}{589} = 0.6548 = 65.48\%$$
Home Problems.
(1) A petrol engine working on the Otto cycle has a compression ratio of 9:1, and at the beginning of compression the temperature is 32°C. After heat energy supply at constant volume, the temperature is 1700°C. The index of compression and expansion is 1.4. Calculate:
(a) temperature at the end of compression;
(b) temperature at the end of expansion;
(c) air standard efficiency of the cycle.
(2) In a diesel cycle the pressure and temperature of the air at the start of compression are 1 bar and 57°C respectively. The volume compression ratio is 16 and the energy added at constant pressure is 1250 kJ/kg. Calculate:
(a) theoretical cycle efficiency;
(b) mean effective pressure.
(3) The swept volume of an engine working on the ideal dual combustion cycle is 0.1068m3 and the clearance volume is 8900cm3. At the beginning of compression the pressure is 1 bar, and temperature is 42°C. If the temperature after expansion is 450°C, the maximum temperature 1500°C and the maximum pressure 45 bar, calculate the air standard efficiency of the cycle.
γ = 1.4,
c_v = 0.715 J/kgK

The indicator diagram

A real-life *p*/*V* diagram is called an *indicator diagram*, which shows exactly what is happening inside the cylinder of the engine. This plot is useful because it allows us to find the work which the engine is doing and therefore its power, and it also enables us to see the effect of the timing of inlet, exhaust and fuel burning, so that adjustments can be made to improve cycle efficiency. In the case of a large slow-speed engine, like a marine diesel engine which typically rotates at about 100 rpm, an indicator diagram can be produced by screwing a device called an engine indicator onto a special cock on the cylinder head of the engine. The indicator records the pressure change in the cylinder and the volume change (which is proportional to crank angle), and plots these on *p*/*V* axes using a needle acting on pressure sensitive paper wrapped around a drum. This produces what is known as an ‘indicator card’.

Figure 10 shows the indicator. The spring in the indicator can be changed to suit the maximum cylinder pressure, so that a reasonable plot can be obtained. Such a mechanical device is not satisfactory for higher-speed engines, but the same result can be plotted electronically.

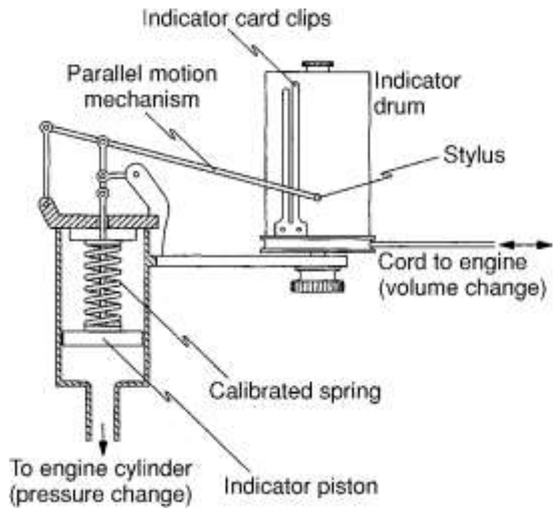

Figure 10 *Engine indicator*

In both cases we get an actual *p*/*V* diagram from within the cylinder, and just as we were able to find work done from our air standard cycles by finding the area within the diagram, so we can find the actual work done, and therefore the power of the engine, by finding the area of the indicator diagram. Of course in this case, the curves are not 'ideal', and the equations cannot be used, but the area within the diagram can be found by some other means, such as by using a planimeter.

Indicated power

As you might expect, the power calculated from the indicator diagram is called the *indicated power* of the engine. It is the power developed inside the cylinder of the engine. As we saw earlier, a value of indicated mean effective pressure can be found by dividing the area of the diagram by its length, but in this case, we must multiply the result by the spring rate of the indicator spring. This gives an 'average' cylinder pressure, used in the expression for indicated power, and it is also used as an important value for comparison between engines.

Indicated power is given by the formula,

$$Indicated\ power, ip = P_{mi}\ A.L.n$$

Where:

$P_{mi} = mean\ effective\ pressure\ \left(\frac{N}{m^2}\right)$

A = area of piston (m^2)

L = length of strike (m)

N = number of power strokes per second.

The verification of this expression can be seen in two ways.

First, we know that the area under the *p*/*V* diagram is work done. The product ($P_{mi}.A.L$) gives this area since P_{mi} is the height of the rectangle and the volume change is given by length multiplied by the area of the bore. The *n* term then imposes a time element which 'converts' the work done to power in kW.

Second, we can use the well-known work done expression from mechanics, work = force × distance. The force on the piston is (pressure × area, i.e. $P_{mi} \times A$), and this force operates over a distance equal to the length of stroke, L. The *n* term then gives power.

Putting the units into our expression for indicated power,

$$ip = \frac{N}{m^2}\ x\ m^2\ x\ m\ x\ \frac{1}{s} = \frac{N.m}{s} = \frac{J}{s} = watts$$

Key points

- The number of power strokes per second is the same as the rev/s for a 2-stroke engine, because there is a power stroke every revolution of the crank.
- For a 4-stroke engine, *n* is the rev/s divided by 2 because there is a power stroke once every two revolutions of the crank.

Brake power

Brake power is the power actually available at the output shaft of the engine. It would be a wonderful world if all the power developed in the cylinders was available at the output shaft, but unfortunately this is not the case because of the presence of friction. This absorbs a certain amount of power, called the friction power. The brake power is, therefore, always less than the indicated power, and this is expressed by the mechanical efficiency of the engine, η_m.

$$\eta_m = \frac{bp}{ip}$$

To find the brake power, it is necessary to apply a braking torque at the shaft by means of a *dynamometer*. The simplest form of this is a rope brake dynamometer which consists of a rope wrapped around the flywheel carrying a load. See Figure 11. More sophisticated types used on high-speed engines are hydraulic or electrical. They all do the same job in allowing the value of braking torque applied to the engine to be measured. This value is put into the formula for rotary power, i.e. $P = T\omega,$ where T is the torque in N.m and ω is the speed of rotation in rad/s. ω can be inserted as $2\pi n,$ since there are 2π radians in one revolution and *n* is the rev/s. We then have the usual form of the equation for brake power,

$$\text{bp} = 2\pi n.T$$

Putting in the units, we have,

$$bp = \frac{1}{s}\ x\ N.m = \frac{N.m}{s}\ \frac{J}{s} = watts$$

Note again that ‘rev’ is dimensionless, as is 2π.

For the rope-brake dynamometer in Figure 2.4.10, the friction load on the flywheel is,

$$(W - S) \text{ newtons}$$

Where W is the applied weight and S is the spring balance reading.

The friction torque is,

$$(W - S)\ \text{x}\ \ r$$

Where r is the radius of the flywheel.

The brake power is then given by,

$$\text{bp} = (W - S)\ \text{x}\ \ r\ x\ \omega \text{ watts}$$

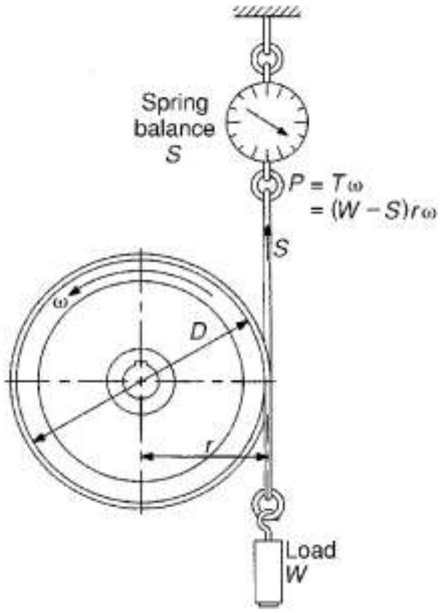

Figure 11 *Rope brake dynamometer*

Key point

When dealing with brake power, remember that we are dealing with the power output from the engine, i.e. from *all* the cylinders combined in a multi-cylinder engine. We usually assume that each cylinder is delivering the same power.

Brake mean effective pressure, P_{mb}

It was explained (see page 36) that a value of *brake mean effective pressure*, P_{mb}, is used as a comparator between engines, because it is easier to find than indicated mean effective pressure, P_{mi}. Brake mean effective pressure is calculated from the indicated power formula with brake power and P_{mb} substituted,

bp = P_{mb} x A x L x n

Fuel consumption

The fuel consumption of an engine is of great importance, and is affected by detail engine design. The figure most often used to express it is a *specific fuel consumption* (*sfc*) based on the number of kg of fuel burned per second for a unit of power output, i.e. the kg of fuel burned per second for each brake kW.

$sfc \ \frac{kg\ fuel\ burned\ per\ sec}{brake\ power\ in\ kW}$, putting in the units,

$sfc = \frac{kg}{s}\ x\ \frac{s}{kJ} = \frac{kg}{kJ}$

An alternative is to express the fuel consumption for each unit of power, e.g for 1 kWh, brake or indicated. A kilowatt hour is a power of 1kW delivered for 1 hour. We then have,

Brake specific fuel consumption,

$bsfc = \frac{kg\ fuel\ burned\ per\ hour}{bp} = \ kg/bkWh$ and,

Indicated specific fuel consumption,

$isfc = \frac{kg\ fuel\ burned\ per\ hour}{ip} = \ kg/ikWh$

These values are also often quoted in grammes, i.e. g/kWh.

Brake and indicated thermal efficiency

The thermal efficiency of the engine can be found by considering, as for all values of efficiency, what we get out for what we put in. In this case we get out a value of brake power and we put in heat energy from the fuel burned. The amount of heat energy we put in is the kg of fuel burned per second multiplied by the calorific value of the fuel, CV in kJ/kg.

If we are using the brake power, the efficiency we get is called the brake thermal efficiency, η_b.

$\eta_b = \frac{brake\ power}{kg\ fuel\ per\ \sec x\ CV}$

which gives units,

$\eta_b = kW\ x\ \frac{s}{kg}\ x\ \frac{kg}{kJ} = 1$

This can be a decimal 0–1, or a percentage.

Indicated thermal efficiency is found in a similar way, i.e.,

$\eta_i = \frac{brake\ power}{kg\ fuel\ per\ \sec x\ CV}$

Example 6

An indicator diagram taken from a large diesel engine has an area of 400mm2 and length 50 mm. The indicator spring is such that the scale of the pressure axis is 1mm = 1 bar. If the cylinder diameter and stroke are both 250 mm and the engine is 4-stroke running at 6 rev/s, find the indicated power if the engine has six cylinders.

Mean effective pressure = P_{mi}

$= \frac{area\ of\ diagram}{length\ of\ diagram}\ x\ spring\ rate$

$= \frac{400}{50}\ x\ 1\ x\ 10^5 = 8\ x\ 10^5\ N/m^2$

Indicated power = $P_{mi}\ A.L.n$

$= 8\ x\ 10^5\ x\ \frac{\pi\ x\ 0.25^2}{4}\ x\ 0.25\ x\ \frac{6}{2} = 29{,}425W\ per\ cylinder.$

Indicated power = ip per cylinder x number of cylinders

= 176 715 W =176.7kW

Example 7

The area of an indicator diagram taken off a 4-cylinder, 4-stroke engine when running at 5.5 rev/s is 390mm2, the length is 70 mm, and the scale of the indicator spring is 1mm = 0.8 bar. The diameter of the cylinders is 150 mm and the stroke is 200 mm. Calculate the indicated power of the engine assuming all cylinders develop equal power.

Solution.

$P_m = \frac{A}{L}\ x\ spring\ rate = \frac{390}{70}\ x\ 0.8 = 4.46\ bar = 4.46\ x\ 10^5\ N/m^2$

$ip = P_m A.L.n\ x\ number\ of\ cylinders$

$= 4.46\ x\ 10^5\ x\ \frac{\pi\ x\ 0.15^2}{4}\ x\ 0.2\ x\ \frac{5.5}{2}\ x\ 4 = 17339W = 17.34kW$

Example 8

During a test, a 2-cylinder, 2-stroke diesel engine operating at 2.75 rev/s records at the dynamometer a brake load of 2.7 kN acting at a radius of 1.6 m. The bore of the cylinder is 0.35m and the stroke is 0.5 m. If the indicated mean effective pressure is 3 bar, calculate:

(a) the indicated power;

(b) the brake power;

(c) the mechanical efficiency.

Solution.

Indicated power = $P_{mi}\, A.L.n$

$= 3\ x\ 10^5\ x\ \frac{\pi\ x\ 0.35^2}{4}\ x\ 0.5\ x\ 2.75\ x\ 2 = 79{,}374W = 79.37kW.$

$bp = T\omega = (2.7\ x\ 1.6)\ x\ 2.75\ x\ 2\pi = 74.64kW$

Note: Torque = force x radius.

$\eta_m = \frac{bp}{ip} = \frac{74.64}{79.37} = 0.94 = 94\%$

Example 9

A marine 4-stroke diesel engine develops a brake power of 3200 kW at 6.67 rev/s with a mechanical efficiency of 90% and a fuel consumption of 660 kg/hour. The engine has eight cylinders of 400 mm bore and 540 mm stroke. Calculate:

(a) the indicated mean effective pressure;

(b) the brake thermal efficiency.

The calorific value of the fuel = 41.86 MJ/kg.

Solution.

$\eta_m = \frac{bp}{ip}, ip = \frac{bp}{\eta_m} = \frac{3200}{0.9} = 3555.6kW$

$ip = P_m A.L.n$

$\frac{3555.6}{8} = P_{mi}\ x\ \frac{\pi\ x\ 0.4^2}{4}\ x\ 0.54\ x\ \frac{6.67}{2}$

$P_{mi} = \frac{3555.6\ x\ 4\ x\ 2}{8\ x\ \pi\ x\ 0.4^2\ x\ 0.54\ x\ 6.67} = 1963.9\, kN/m^2 = 19.63\ bar.$

$Brake\ thermal\ efficiency = \frac{brake\ power}{kgfuel/s\ x\ CV}$

$= \frac{3200}{\frac{660}{3600}\ x\ 41.86\ x\ 10^3} = 0.417 = 41.7\%$

Example 10

A 6-cylinder 4-stroke internal combustion engine is run on test and the following data was noted:

Compression ratio = 8.2:1 Speed = 3700 rpm

Brake torque = 0.204 kN.m Bore = 90mm

Fuel consumption = 26 kg/h Stroke = 110mm

Calorific value of fuel = 42 MJ/kg

Indicated mean effective pressure = 7.82 bar

Calculate:

(a) the mechanical efficiency;

(b) the brake thermal efficiency;

(c) the brake specific fuel consumption.

Solution.

ip = $P_{mi} A.L.n$ x number of cylinders.

$= 782\ x\ \pi\ x\ \frac{0.09^2}{4}\ x\ 0.11\ x\ \frac{3700}{120}\ x\ 6 = 101.2kW$

$bp = T\omega = 0.204\ x\ \frac{3700\ x\ 2\pi}{60} = 79kW$

$\eta_m = \frac{bp}{ip} = \frac{79}{101.2} = 0.781 = 78.1\%$

$\eta_b = \frac{bp}{kgfuel/s\ x\ CV} = \frac{79}{\frac{26}{3600}\ x\ 42\ x\ 10^3} = 0.26 = 26\%$

$Brake\ specific\ fuel\ \ consumption(bsfc) = \frac{kgfuel/h}{brake\ power} = \frac{26}{79} = 0.329\, kg/kWh$

Volumetric efficiency

The *volumetric efficiency* of an engine – or a reciprocating compressor- is a measure of the effectiveness of the engine in ‘breathing in’ a fresh supply of air.

Under perfect circumstances, when the piston starts to move from top dead centre down the cylinder, fresh air is immediately drawn in. However, above the piston at TDC there is a residual pressure which remains in the cylinder until the piston has moved down the cylinder a sufficient distance to relieve it and create a pressure slightly below atmospheric. Only then will a fresh charge of air be drawn in. A further difficulty is the heating of the air in the hot inlet manifold, which also reduces the mass of air entering the cylinder.

The ratio of the swept volume of the engine to the volume of air actually drawn in is called the volumetric efficiency, η_v.

$$\eta_v = \frac{volume\ of\ change\ induced\ at\ reference\ temperature\ and\ pressure}{piston\ swept\ volume}$$

The reference temperature and pressure are usually the inlet conditions.

Example 11

A 4-stroke, 6 cylinder engine has a fuel consumption of 26 kg/h and an air/fuel ratio of 21:1. The engine operates at 3700 rpm and has a bore of 90 mm, stroke 110 mm. Calculate the volumetric efficiency referred to the inlet conditions of 1 bar, 15°C. R = 287 J/kgK.

Solution.

Using the characteristic gas equation, $p_1V_1 = m.R.T_1$

Volume of air induced/minute

$$= \frac{m.R.T_1}{P_1} = \frac{\frac{(2x\ 216\ )}{60}\ x\ 287\ x\ (15+273)}{1\ x\ 10^5} = 7.52m^3/min.$$

$$Swept\ volume = \frac{\pi\ x\ 0.09^2}{4}\ x\ 0.11 = 7\ x\ 10^{-4}\ m^3/rev$$

$$= 7\ x\ 10^{-4}\ x\ \frac{3700}{2}\ x\ 6 = 7.76\ m^3/min$$

$$\eta_v = \frac{volume\ of\ change\ induced\ at\ reference}{swept\ volume} = \frac{7.52}{7.76} = 0.97 = 97\%$$

If, given a volume, you need to change it to a different set of conditions, use can be made of

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

CASE STUDY

Marine diesels

Diesel engines are produced by many manufacturers, in a range of power outputs, for very many applications. The largest diesel engines are to be found in ships, and these operate on the 2-stroke cycle, which makes them quite unusual. The piston is bolted to a piston rod which at its lower end attaches to a crosshead running in vertical guides, i.e. a crosshead bearing. A connecting rod then transmits the thrust to the crank to turn the crankshaft. The arrangement is the same as on old triple expansion steam engine, from which they were derived. They have the further peculiarity of being able to run in both directions by movement of the camshaft. This provides astern movement without the expense of what would be a very large gearbox. These very large engines are the first choice for most merchant ships because of their economy and ability to operate on low quality fuel. A typical installation on a container ship, for instance, would be a 6-cylinder turbocharged engine producing 20 000Kw at a speed of about 100 rpm. The engine is connected directly to a fixed-pitch propeller. Most diesel engines are now turbocharged. Exhaust gas from the engine drives a gas turbine connected to a fan compressor which forces air into the cylinder at a raised pressure. This has the main advantage of charging the cylinder with a greater mass of air (the mass is proportional to the pressure, from $pV = m.R.T$), thereby allowing more fuel to be burned, so for the same size cylinder more power can be produced. An added advantage in the case of a 2-stroke engine is that by pressurizing the air into the cylinder, the exhaust gas is more effectively removed or 'scavenged' before the next cycle begins One of the main problems with large slow-speed engines is the headroom necessary to accommodate them, and in a vessel such as a car ferry, they are not usually fitted because they would limit car deck space. Instead, medium-speed engines are used which are 4-stroke and are of the more usual trunk-piston configuration, the same as a car engine and almost all other engines too. One of the latest engines, developed for fast ferries, has the following particulars:

Power output 8200kW

Operating cycle 4-stroke

Number of cylinders 20, in 'V' configuration

Bore 265mm

Stroke 315mm

Operating speed 1150 rpm

Dimensions 7.4 m long × 1.9 m wide × 3.3 m high

Weight 43 tonnes (43 000 kg)

Mean effective pressure 24.6 bar

Specific fuel consumption 195 kg/kWh

Time between overhauls 24 000 hours

The engine has a single large turbocharger at one end. Clearly, this is a sizeable engine, and typically a large ferry would need two or three of them. Most cruise ships also have these 'medium speed' diesel engines. Many manufacturers produce a single engine design in which the number of cylinders in the complete engine can be varied to suit the required output. This simplifies spares and maintenance requirements and means that the engine builder can tailor an engine of a standard design to meet different requirements.

The details below illustrate this for an engine type now in production. Note the number of variations which can be obtained and therefore the range of power outputs available:

Operating cycle 2-stroke

Bore 350mm

Stroke 1400mm

Number of cylinders 4, 5, 6, 7, 8, 9, 10, 11 or 12

Power output 2900–8900kW

Mean effective pressure 19 bar

Fuel consumption 180 g/kWh